ADMISSIONS TEST

MSc in Financial Mathematics
University of Warwick

The questions below check that you have sufficient background in mathematics to do the MSc in Financial Mathematics at the University of Warwick.

You are allowed to use the texts cited at the end of the test. However, make sure that after revising the material you can do this test (or a similar test) without using any material.

1 Linear algebra

1. Find an orthogonal basis for the subspace of $\mathbb{R}^4$ spanned by three vectors
$$(1,1,1,1),\ (2,1,2,1),\ (0,1,0,1).$$

2. Let A be a matrix (not necessarily square). Prove that all eigenvalues of the matrix A^*A are non-negative real numbers. (A^* denotes the conjugate transpose matrix of A).

3. Let A, B be square matrices of the same size. Prove that
$$tr(AB) = tr(BA).$$
(tr denotes the trace of a matrix)

4. Do there exist 3×3 matrices A, B such that
$$AB - BA = \begin{pmatrix} 1 & -2 & 6 \\ 2 & 0 & -1 \\ -6 & 1 & 1 \end{pmatrix}?$$
(Hint: Use the result of the previous exercise)

2 Analysis

1. Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable L_1 function and let $\tilde{f}$ be its Fourier transform. What is Fourier transfer of the function $f(x + c)$, where $c \in \mathbb{R}$ is a constant? What is Fourier transfer of the derivative of f?

2. Let $a, b, c > 0$ and let y be the solution of the following ODE

$$a\frac{d^2y}{dt^2} + b\frac{dy}{dt} + cy = 0, \quad y(0) = c_1, \quad \frac{dy}{dt}(0) = c_2.$$

   Show that

$$\lim_{t\to+\infty} y(t) = 0.$$

3. Assume a solution of the form $u(x, t) = X(x)T(t)$ to the modified diffusion equation $u_t - Du_{xx} - \alpha u = 0$. First show that the equation separates and find the general solution for $X(x)$ and $T(t)$. Next, assuming that $D > 0$, $\alpha \geq 0$, $L > 0$, solve the boundary value problem

$$\begin{aligned} u_t - Du_{xx} - \alpha u &= 0, \qquad \text{for all } 0 \leq x \leq L,\ t \geq 0 \\ u_x(0, t) = u_x(L, t) &= 0, \qquad \text{for all } t \geq 0 \\ u(x, 0) = \cos(\pi x/L) &+ \cos(2\pi x/L). \end{aligned}$$

3 Probability

1. The joint distribution of X and Y is given by $f(x, y) = e^{-y}/y$, $0 < x < y < \infty$. Compute $E(X^2 + Y^2 \,|Y = y)$.

2. Let X be a random variable. The *cumulants* κ_n of X are defined by the cumulant-generating function

$$g(t) = \log(E(\exp(tX))).$$

   The n^{th} cumulant is given by $\kappa_n = g^{(n)}(0)$ (the n^{th} derivative of g at 0).

   Find the first five cumulants of the normal one dimensional distribution.

3. In a game of chess there are 32 pieces, 4 of which are Bishops. If you select the pieces at random, without replacement, and place them in a line, what is the probability that there are no Bishops next to each other?

4 Another recommendation

Apart from being able to solve the above questions, it is strongly suggested that you read some book on finance. The book M Baxter and A Rennie cited below is an excellent start. Other books are listed on the financial maths website

`http://www2.warwick.ac.uk/fac/cross_fac/financial_maths/faqs/`

5 References

1. Linear algebra

   - G. Strang, *Linear Algebra and its Applications*, Academic Press 1980, ISBN 012673660X
   - K. Hoffman and R. Kunze, *Linear Algebra*, Prentice Hall 1971, ISBN 0135467972

2. Probability Theory

   - G. Grimmett, *Probability: an Introduction*, Clarendon 1986, ISBN 9198532725
   - S.M. Ross, *Introduction to Probability Models*, Academic Press 1993, ISBN 0125984553

3. Analysis

   - W.E. Boyce, *Elementary Differential Equations and Boundary value Problems*, Wiley 1997, ISBN 0471089559

4. Finance

   - M. Baxter and A. Rennie, *Financial Calculus* (in particular chapters 1 to 4), Cambridge University Press 1996, ISBN 0521552893